Univerza v Ljubljani

Univerza v Ljubljani

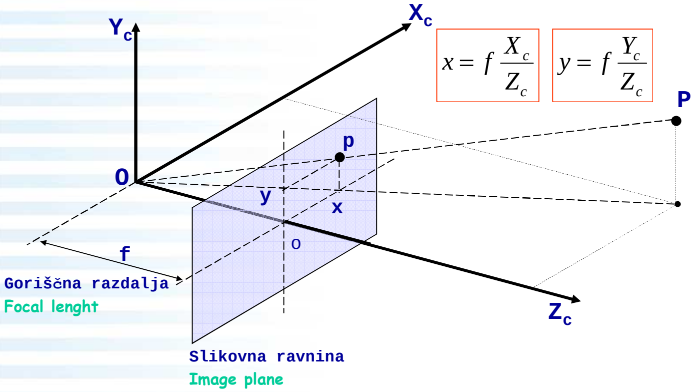
Fakulteta za elektrotehniko

Modeliranje kamere #2

Stanislav Kovačič

http://vision.fe.uni-lj.si/

Perspektivna projekcija



$$x = f\frac{X_c}{Z_c} \qquad y = f\frac{Y_c}{Z_c}$$

Iz vsebine

- Nastanek slike, osnovno o modeliranju kamere
  - Image formation, basics of camera modeling
- Direktna linearna transformacija (DLT)
  - Direct Linear Transform (DLT)
- Kalibracija kamere (DLT, Tsai)
  - Calibration (DLT, Tsai)
- Rekonstrukcija – “nazaj v prizor”
  - Reconstruction - back from 2D to 3D
- Še nekaj pogledov na modeliranje kamere
  - Camera model revisited
- Distorzija leče
  - Lens distortion

Calibration matrix

$$x = f\frac{X_c}{Z_c}$$

$$y = f\frac{Y_c}{Z_c}$$

Intrinsic matrix

$$\Rightarrow \quad w\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

Homogene koordinate

Homogeneous coordinates

$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ \frac{w}{w} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x/w \\ y/w \end{bmatrix}$$

Calibration matrix

Slikovne diskretizirane koordinate

Discretized image coordinates

$\lambda_u(u-u_0)=x$

$\lambda_v(v-v_0)=y$

$\Rightarrow$

Image center

$$w\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}f_u & 0 & u_0 & 0\\ 0 & f_v & v_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}X_c\\Y_c\\Z_c\\1\end{bmatrix}$$

Pixel size

Projection matrix

Zunanji koordinatni sistem = Koordinatni sistem kamere

World coordinate system = Camera coordinate system

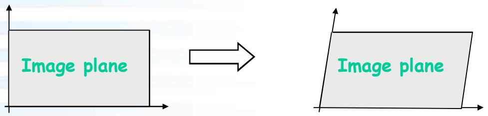
$$w\begin{bmatrix}u\\v\\1\end{bmatrix}=\mathbf{K}\begin{bmatrix}X_c\\Y_c\\Z_c\\1\end{bmatrix}=\mathbf{K}[\mathbf{I}\quad\mathbf{0}]\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}=\mathbf{K}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

Calibration matrix

Slikovne diskretizirane poševne koordinate

Discretized and skewed image coordinates

Additional parameter: skew



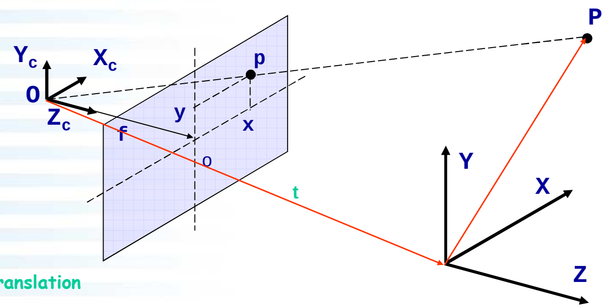
$$w\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}f_u & \gamma & u_0 & 0\\ 0 & f_v & v_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}X_c\\Y_c\\Z_c\\1\end{bmatrix}=\mathbf{K}\begin{bmatrix}X_c\\Y_c\\Z_c\\1\end{bmatrix}$$

K - calibration matrix / intrinsic parameters matrix

Projection matrix

Zunanji koordinatni sistem ≠ Koordinatni sistem kamere

World coordinate system ≠ Camera coordinate system



Projection matrix

Translation t

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \mathbf{K}\begin{bmatrix}X_c\\Y_c\\Z_c\\1\end{bmatrix} = \mathbf{K}[\mathbf{I} \quad \mathbf{t}]\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix} = \mathbf{K}\begin{bmatrix}1 & 0 & 0 & t_x\\0 & 1 & 0 & t_y\\0 & 0 & 1 & t_z\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

Extrinsic matrix

Projection matrix

Rotation, translation

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \mathbf{K}\begin{bmatrix}X_c\\Y_c\\Z_c\\1\end{bmatrix} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}]\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix} = \mathbf{K}\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\r_{21} & r_{22} & r_{23} & t_y\\r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

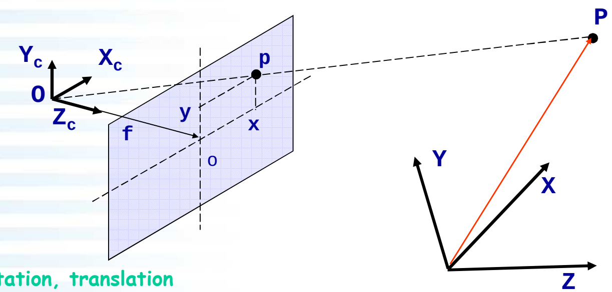
R – Rotation

t - Translation

Projection matrix

Zunanji koordinatni sistem ≠ Koordinatni sistem kamere

World coordinate system ≠ Camera coordinate system



Rotation, translation

Projekcijska matrika

M – projection matrix

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}m_{11} & m_{12} & m_{13} & m_{14}\\m_{21} & m_{22} & m_{23} & m_{24}\\m_{31} & m_{32} & m_{33} & m_{34}\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

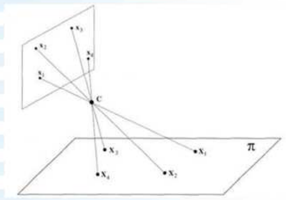
Matrix (de)composition

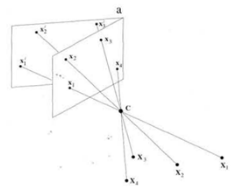
$$M = M_{int} M_{ext}$$

$$M_{int} = \begin{bmatrix} f/\lambda_u & \gamma & u_0 \\ 0 & f/\lambda_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

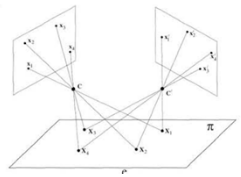
$$M_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}$$

Plane to plane mapping





R. Hartley, A. Zisserman
Multiple View Geometry in Computer Vision
Second Edition, 2003



Plane to plane mapping

$Z=0$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{K}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} \qquad w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

H - homography (projective transformation)
3 x 3 matrix, full rank, invertible, 8 DoF

$$\mathbf{H}_{3\times3} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

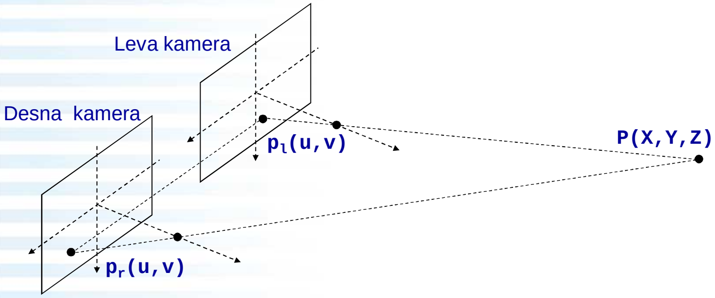
$$w\mathbf{u} = \mathbf{H}\mathbf{p}$$

Each point pair contributes 2 equations, thus 4 point are sufficient to solve for H.

Epipolar geometry

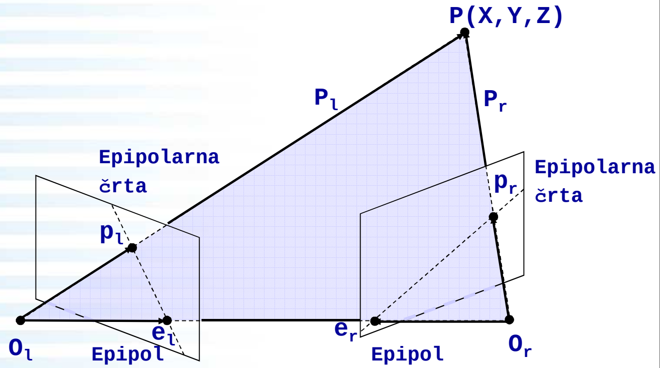
- Stereopsis (stereovid)
- Epipolar plane, epipolar lines, epipoles
- Essential matrix (bistvena matrika)
- Funamental matrix (fundamentalna matrika)

Stereo – osnovni model



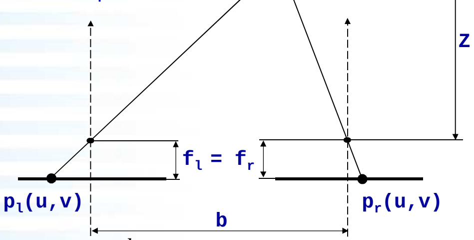
Disparity - Dispariteta (neskladje), $p_l(u,v) \neq p_r(u,v)$ je osnova za izračun razdalje do točke v prostoru.

Epipolarna geometrija



Stereo – lateralni model

- Optični osi sta vzporedni
- Koordinatne osi so poravnane



$$Z = f\frac{b}{d}, \quad d = u_r - u_l, \quad v_r = v_l$$

d: stereo dispariteta

Epipolarna geometrija

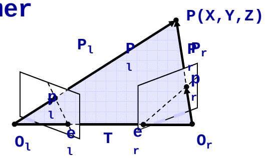
Pomen epipolarne geometrije:

- 1D search - omeji prostor iskanja (2D -> 1D): dani točki v eni sliki pripada (korespondenčna) točka, ki leži na epipolarni črti.
- false matches - zmanjša število "lažnih" ujemanj, ker je potencialnih kandidatov manj!

Epipolarna geometrija

Preslikava med k.s. kamer

$$P_r = R(P_l - T)$$



Preslikava v slikovno ravnino

$$p_r = \gamma_r P_r = \frac{f_r}{Z_r} P_r$$

$$p_l = \gamma_l P_l = \frac{f_l}{Z_l} P_l$$

Epipolarna geometrija

Projekcija v diskretno slikovno ravnino

$$\mathbf{P}_r^T \mathbf{E} \mathbf{P}_l = 0$$

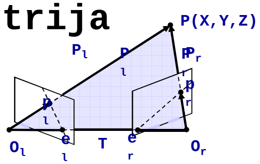
$$\mathbf{p}_r^T \mathbf{K}_r^{-T} \mathbf{E} \mathbf{K}_l^{-1} \mathbf{p}_l = 0$$

Projekcijski matriki (notranji parametri)

$$\mathbf{p}_r^T \mathbf{F} \mathbf{p}_l = 0$$

Matriko F določajo zunanji in notranji parametri sistema

Epipolarna geometrija



Enačba epipolarne ravnine

$$(\mathbf{P}_l - \mathbf{T})^T (\mathbf{T} \times \mathbf{P}_l) = 0$$

$$(\mathbf{R}^T \mathbf{P}_r)^T (\mathbf{T} \times \mathbf{P}_l) = 0$$

$$\mathbf{P}_r^T \mathbf{R} (\mathbf{T} \times \mathbf{P}_l) = 0 \qquad \text{Op.: } \mathbf{T} \times \mathbf{P}_l = \mathbf{S}\mathbf{P}_l = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix} \mathbf{P}_l$$

$$\mathbf{P}_r^T \mathbf{R} \mathbf{S} \mathbf{P}_l = 0$$

$$\mathbf{P}_r^T \mathbf{E} \mathbf{P}_l = 0$$

Matriko E določajo zunanji parametri sistema

Stereo kalibracija

Določanje fundamentalne matrike

$$\mathbf{p}_r^T \mathbf{F} \mathbf{p}_l = 0$$

Vsaka točka prispeva eno enačbo
Potrebujemo (najmanj) 8 točk

Spet optimizacijski problem:
- Metoda najmanjših kvadratov
- SVD

3-D rekonstrukcija

- Poznamo zunanje in notranje parametre:
  3-D rekonstrukcija je možna (triangulacija)

- Poznamo notranje parametre:
  3-D rekonstrukcija in določitev zunanjih parametrov
  je možna do konstante (faktorja skaliranja) natančno.

- Poznamo samo korespondenčne točke:
  rekonstrukcija je možna samo do projektivne
  transformacije natančno.

Literatura

- E. Trucco, A. Verri, Introductory Techniques for 3D Computer Vision, Prentice Hall, 1998.
- M. Sonka, V. Hlavač, R. Boyle, Image Processing, Analysis, and Machine Vision, Thomson, 2008.
- Z. Zhang, “A flexible new technique for camera calibration”, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 22(11):1330–1334, 2000.